Letters

Comments on "Internal Impedance of Conductors of Rectangular Cross Section"

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***Abstract*—For rectangular conductors, as found in planar transmission lines, resistance and inductive reactance deviate by some 10%, in contrast to common approximations assuming equality of both quantities. This paper discusses this effect and presents quantitative data for the practical case of a monolithic-microwave integrated-circuit coplanar waveguide.**

I. Motivation

The above paper,[1] reports an interesting observation, namely, that for conductors of rectangular cross section, the internal inductive reactance ωL_i in the skin-effect regime is 20%–30% lower than the resistance R. This is in contrast to the case of circular cross section and the one-dimensional Cartesian approximation where the quantities R and ωL_i are equal. Strictly speaking, this is not surprising because the simple skin-effect relations including $R = \omega L_i$ apply whenever the curvature radius of a surface is large compared to skin depth, which causes a considerable deviation from the simple rule in conductors with pronounced edge current contributions. Nevertheless, the issue is of importance since, in many cases, the assumption of equal reactance and resistance is used not only in its strict sense. Instead, it forms the basis of several skin-effect descriptions, which are commonly applied to rectangular conductors as well, e.g., Wheeler's incremental inductance rule [1]. Therefore, further investigations are necessary to clarify this subject.

II. Method of Analysis

In the course of our research on conductor loss in planar transmission lines [2], [3], we have also dealt with the internal inductance topic. In the following, recent results on coplanar structures are presented that supplement the data in the above paper. The mode-matching approach of [2] is used for analysis. The considerations focus on coplanar transmission lines of monolithic-microwave integrated-circuit (MMIC) typical geometry, which require a detailed description of conductor-loss phenomena. Two different geometries are treated (see Fig. 1), i.e., a 50-Ω coplanar waveguide (CPW) with 20-μm center conductor and 50-μm ground-to-ground spacing, and, as a special structure highlighting the skin-effect phenomena, a single 20-μm-wide metal strip surrounded by a lateral electric wall.

The full-wave mode-matching approach [2] employs a rigorous description of conductor loss without any *a priori* assumptions regarding skin effect. An eigenvalue problem yields the complex propagation constants and the corresponding mode fields from which all other quantities can be obtained. Internal inductance and resistance of the conductors are calculated from the current and the stored magnetic energy and dissipated power, respectively.

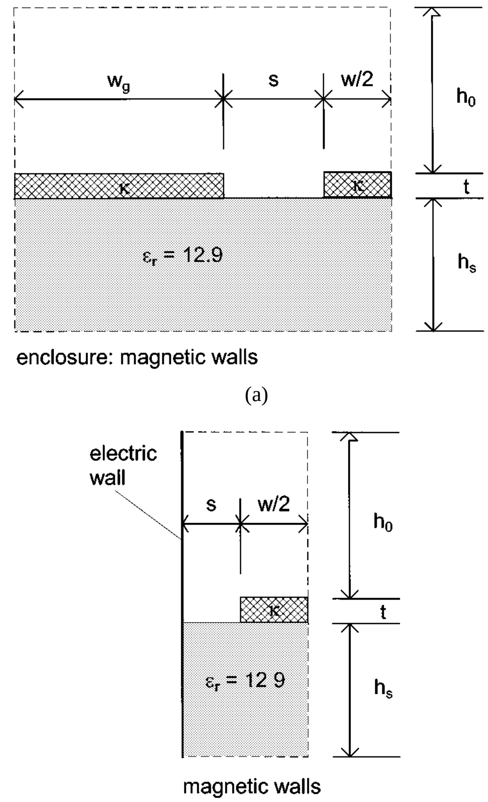

Fig. 1. Two structures under investigation. (a) CPW (left-half cross section, enclosed by magnetic walls) with $w = 20$ μm, $s = 15$ μm, $t = 3$ μm, and 6 μm. Substrate thickness: $h_s = 100$ μm, $\varepsilon_r = 12.9$, $h_0 = 500$ μm. Conductivity of metallizations $\kappa = 30$ S/μm. (b) Related single-conductor geometry with $w = 20$ μm, $s = 7.5$ μm, $t = 6$ μm, and left-hand-side electric wall ($h_s = h_0 = 50$ μm, $\varepsilon_r = 12.9$).

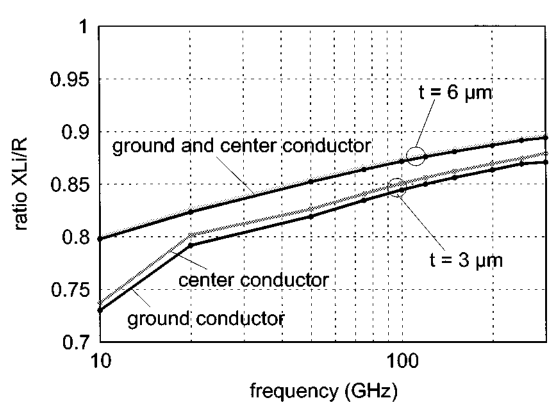

Fig. 2. CPW according to Fig. 1(a). Ratio between internal reactance $XL_i = \omega L_i$ and resistance R for the center and ground conductors as a function of frequency (mode-matching method [2]).

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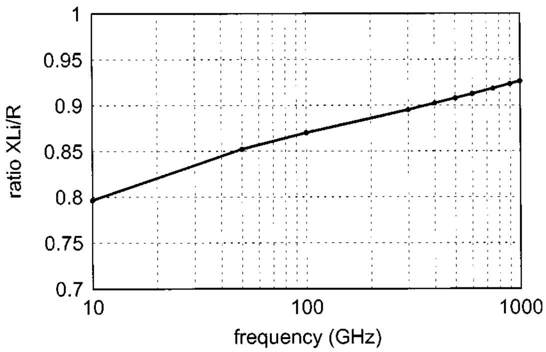

Fig. 3. Structure of Fig. 1(b). Ratio between internal reactance $XL_i = \omega L_i$ and resistance R as a function of frequency.

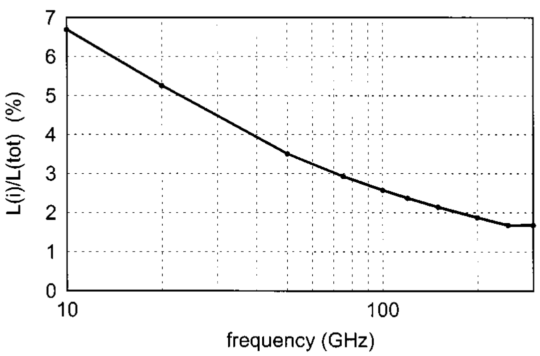

Fig. 4. CPW of Fig. 1(a). Relation $L_i/L_{\rm tot}$ between sum of internal inductances and total line inductance in percent against frequency (metal thickness $t = 6$ μm).

III. Results

What is of interest here is the ratio between internal inductive reactance $XL_i = \omega L_i$ of each conductor and the corresponding resistance R. Fig. 2 presents the results for the CPW in Fig. 1(a) with metal thicknesses t of 3 and 6 μm, respectively. The lower frequency limits for the skin-effect range ($\delta = t/3$) are 8.4 and 2.1 GHz, respectively, which is below the frequency range shown in Figs. 2 and 3.

In accordance with the above paper, internal reactance XL_i is lower than resistance R. Quantitatively, however, the ratio XL_i/R is closer to one than found in the above paper. Also, there appears to be a monotonic increase with frequency. Note that, in Fig. 2, no saturation is detected. In order to probe further, we analyzed the structure in Fig. 1(b), which consists of the CPW center conductor in a different environment allowing quasi-TEM propagation up to higher frequencies than the CPW in Fig. 1(a). The results are plotted in Fig. 3. The data up to 300 GHz is almost equal to that in Fig. 2. This proves that indeed L_i and R depend primarily on the conductor geometry and not on the environment. However, as in the previous plot, the curve increases steadily without any indication of saturation up to the highest frequency. At 1 THz, which corresponds to $\delta/t = 0.015$, XL_i amounts to 93% of R (the aspect ratio w/t of the conductor is $20/6 = 3.33$).

IV. Conclusions

- For conductors of rectangular cross section, the internal inductive reactance $XL_i = \omega L_i$ deviates considerably from the resistance R, even if thickness t is much larger than skin depth δ. This supports the finding in the above paper.
- However, the difference, which is about 13% at $\delta = t/20$, does not saturate, but decreases monotonically with growing frequency. Thus, equality between ωL_i and R will presumably be reached in the ultimate limit $f \to \infty$, though the deviations decrease slowly and, for most practical cases, a deviation of some 10% remains. This is probably caused by the field characteristics at the edges, whose relative influence decreases with vanishing skin depth, though at a slow rate.
- When assessing the importance of the above-mentioned inequality, one has to relate the deviations of the internal inductance to the total line inductance. As an example, in Fig. 4, the percentage ratio between internal inductances and total inductance for the CPW, according to Fig. 1(a), is plotted as a function of frequency. The sum of internal inductances amounts to less than 10% of the total line inductance. This means: a 10% uncertainty in L_i translates into less than 1% error in line inductance. Therefore, the relevance to practical circuit design is limited and it needs highly accurate analysis tools to investigate these effects.

References

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